The many sources effect on the genuine multihadron correlations *

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Here we report on a study aimed to explore the dependence of the genuine multiparticle correlations on the number of sources while the influence of other possible factors affecting the multihadron production is avoided. The analysis utilised the normalised cumulants, calculated in three-dimensional phase space, of the reaction $e^+e^- \to Z^0 \to$ *hadrons* using a large Monte Carlo event sample. The multi-sources reactions were simulated by overlaying a few independent single e^+e^- annihilation events. It was found that as the number of sources S increases, the cumulants do not change significantly their structure, but those of an order $q > 2$ (i.e. more than 2 pions) decrease fast in their magnitude. This reduction can be understood in terms of combinatorial considerations of source mixing which dilutes the correlations by a factor of about $1/S^{q-1}$ which can also serve as a method to estimated the number of sources. This expected suppression is well reproduced by recent cumulant measurements in hadron and nucleus induced reactions both in one (rapidity) and two (rapidity vs. azimuthal angle) dimensions. The diminishing genuine correlations effect should also appear in other dynamical correlations like the Bose-Einstein in $e^+e^- \to W^+W^- \to$ *hadrons* and in nucleus-nucleus reactions.

1. INTRODUCTION

During the last decade an increase interest has been shown for the genuine multiparticle correlations in multihadron final states of hadronic, e^+e^- and other reactions [1]. Recently OPAL, in its study of e^+e^- annihilations on the Z^0 mass, has established the existence of strong genuine multihadron correlations up to the fifth order [2]. In hadron-hadron, like proton-proton, collisions the correlations of more than three particles have also been observed [3–5]. In contrast to this situation, in heavy ion collisions, at low energies and/or in reactions of light nuclei, genuine correlations are found to have non-zero values only up to the third order [6]. Furthermore it has been found out that in general these correlations become weaker as the reaction average multiplicity increases. In nucleus-nucleus collisions at high energies, of tens and hundreds GeV per nucleon, the two-particle correlations are the only one that survive [3, 7–9].

This correlation dependence on the average multiplicity is very similar to the one observed in the investigations of multiparticle dynamical fluctuations, i.e. variation of many particle bunches in restricted phase space regions [1]. In these studies, known as intermittency analyses, the observed average multiplicity dependence of the correlations has been proposed to be the consequence of a mixing of several independent emission sources [8, 10, 11]. As a result, the dynamical fluctuations in nucleus-nucleus collisions are already well accounted for by two-particle correlations [7, 12], whereas in hadron-hadron interactions [4, 13] and in e^+e^- annihilations [2] higher order genuine correlations do exist.

An analogous situation seems also to exist in the Bose-Einstein correlations (BEC) where identical bosons are correlated when they emerge from the interaction in nearby phase space. A genuine three-pion BEC has been detected in hadron-hadron reactions [14] and found to be even more pronounced in e^+e^- annihilations [15]. On the other hand the NA44 collaboration [16] has not found, in their study of sulphur-lead collisions at 200 GeV/A, any genuine three-pion BEC as the three-body correlations were well reproduced in terms of two-particle BEC. In a recent reprint, the

*Invited talk presented by G. Alexander at the 9th International Workshop on Multiparticle Production, Turin, Italy, June 12 - 17, 2000.

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WA98 collaboration [17] reported on their study of central lead-lead collisions at 158 GeV/c per nucleon where an evidence for a three-pion BEC seems to exist. However it was pointed out that, at least some of the BEC analyses carried out with the super proton synchrotron at CERN, may have to be reevaluated [18]. Since the intermittency phenomenon and BEC seem to be closely related [1], the dependence of many sources on the strength of the BEC cannot be excluded. The superposition of emitters may also be a reason for the suppression of BEC of hadrons produced from W-boson pairs in e^+e^- annihilations at LEP2 energies where the overlapping of hadrons affect the accuracy of the W mass measurements [19].

All this, as well as the obvious intrinsic interest in the genuine correlations which carry most of the dynamics of the hadron production process, points to the need of dedicated studies aimed to investigate the correlation dependence on the number of emission sources. Here we study this dependence by grouping several $e^+e^- \rightarrow$ *hadrons* events to represent a multi-emission sources of particles. To obtain significant results, even when only few sources are considered, one needs a very high statistics, like that which can be supplied by a Monte Carlo (MC) generated sample. This allows to minimise the calculation error and thus be sensitive to the correlations dependence on the number of sources. Another advantage of using MC generated events is in its possibility to generate a multihadron sample free from contamination of other processes like, for example, $e^+e^- \rightarrow \tau^+\tau^- \rightarrow$ *hadrons*.

Here we present the results of a MC study on the effect of several emission sources on the genuine higher order multiparticle correlations [20]. The study was based on a generated sample of about 5×10^6 events of the reaction $e^+e^- \rightarrow Z^0 \rightarrow$ *hadrons* which passed a full simulation of the OPAL detector at LEP and did reproduce rather well the measured genuine high order correlations present in the OPAL hadronic Z^0 decay data [2]. Moreover, the $e^+e^- \rightarrow Z^0 \rightarrow$ *hadrons* annihilations should represent well the one emission source situation in contrast to events produced in hadron-hadron interactions. Our analysis on the dependence of many sources on the genuine correlations was thus carried out in a way that avoided effects coming from other features, like the multiplicity which was discussed recently in connection with two-particle BEC analyses in [21] and [22]. Inasmuch that final state interactions between hadrons coming from different sources can be neglected, our correlation study based on $e^+e^- \rightarrow Z^0 \rightarrow$ *hadrons* annihilations may be extended to other types of reactions since the hadronisation process is believed to rest on a common basis [23].

2. THE ANALYSIS METHOD

The analysis is based on a generated sample of hadronic Z^0 decays using the JETSET 7.4 MC program [24] including a full simulation of the OPAL detector at LEP [25]. The MC sample also included initial-state radiation and effects of finite lifetimes. The parameters of the program were tuned to yield a good description of the measured event shape and single particle distributions [26].

The selection criteria for multihadron events used here are identical to the ones previously utilised by OPAL in their recent data analysis of multiparticle correlations [2]. In particular, selected events were required to have at least five charged tracks each having at least 20 measured points in the jet chamber where the first point had to be closer than 40 cm from the beam axis. The cosine of the polar angle of the event sphericity axis with respect to the beam direction was required to be less than 0.7 to ensure that the event lies within the volume of the detector. The sphericity axis was calculated by using all accepted tracks and electromagnetic and hadronic calorimeter clusters.

To simulate several emission sources we did overlay several $e^+e^- \rightarrow Z^0 \rightarrow$ *hadrons* generated events and analysed the correlations between pions as if they were created in a single event. The kinematic variables are defined within each generated e^+e^- event with respect to its own sphericity axis. For correlation analyses of variables like rapidity this procedure is equivalent to the one where the events are rotate to a common sphericity axis. This simulates multi-sources' events producing hadrons, here taken to be pions, emerg-

ing from a common emitter. To note is that in this procedure the average event multiplicity is directly proportional to the number of sources. In our analysis each generated event was used only once to avoid correlations between different sources, which for this reason required a very large MC event sample.

To extract the genuine dynamical q-particle correlations, we used bin-averaged normalised factorial cumulant moments, or cumulants, first proposed in Ref. [27] as a tool for the search of genuine multiparticle correlation,

$$K_q = \frac{1}{M}\sum_{m=1}^{M}\int_{\delta y}\prod_i \mathrm{d}y_i \frac{C_q(y_1,\ldots,y_q)}{[\int_{\delta y}\mathrm{d}y\rho_1(y)]^q}\,. \qquad (1)$$

The $C_q(y_1,\ldots,y_q)$ are the q-particle correlation functions given by the inclusive q-particle density distributions $\rho_q(y_1,\ldots,y_q)$ in terms of cluster expansion, e.g.,

$$\begin{aligned} C_3(y_1,y_2,y_3) &\\ &= \rho_3(y_1,y_2,y_3) - \sum_{(3)}\rho_1(y_1)\rho_2(y_2,y_3) \\ &\quad + 2\,\rho_1(y_1)\rho_1(y_2)\rho_1(y_3)\,. \qquad (2)\end{aligned}$$

Here M is the number of equal bins, having a width δy, into which the event phase-space is divided and the subscript (3) denotes the number of permutations. For simplicity we show all formulae in 1-dimensional (e.g., rapidity) phase space.

The feature of the C_q-functions is that they vanish whenever there are no genuine correlations, i.e., the correlations are due to those present in lower orders. The correlations extracted are of a dynamical nature since the cumulants share with normalised factorial moments (the intermittency analysis tool) the property of statistical noise suppression.

Here we computed the cumulants as they are used in experimental studies, in particular we used the form applied in Ref. [2] namely,

$$K_q = \frac{\mathcal{N}^q\cdot\sum_{m=1}^{M}k_q^{(m)}}{\sum_{m=1}^{M}N_m(N_m-1)\cdots(N_m-q+1)}\ . \qquad (3)$$

The $k_q^{(m)}$ factors are the unnormalised factorial cumulant moments, or the Mueller moments [28], calculated for the mth bin. These factors represent the correlation functions C_q integrated over the bin and N_m is the number of particles in the mth bin summed over all the $\mathcal{N}$ events. The definition (3) takes into account the non-uniform shape of the single-particle distribution and the bias when the cumulants are computed at small bins.

The cumulant calculations were performed in the three-dimensional phase space of the kinematic variables commonly utilised in this kind of studies [1], namely:

- The rapidity, $y = \ln\sqrt{(E+p_\parallel)/(E-p_\parallel)}$, with E and $p_\parallel$ being the energy and longitudinal momentum of the hadron within the interval $-2.0 \le y \le 2.0$;
- The transverse momentum in the interval $0.09 \le p_T \le 2.0$ GeV/c;
- The azimuthal angle, $0 \le \Phi < 2\pi$, calculated with respect to the eigenvector of the momentum tensor having the smallest eigenvalue, in the plane perpendicular to the sphericity axis.

These variables are defined with respect to the sphericity axis, in a way and within the intervals similar to those used in a recent OPAL analysis [2] and in other cumulant studies [1].

3. GENUINE CORRELATIONS AND THE NUMBER OF SOURCES

3.1. Monte Carlo studies

In Fig. 1 we reproduce the cumulants of orders $q = 2$, 3 and 4 measured by OPAL and compared with those based on two MC models, Jetset 7.4 and Herwig 5.9 [2]. The cumulants were calculated in 1-dimensional sub-space of rapidity, in 2-dimensional rapidity vs. azimuthal angle sub-space and in 3-dimensional phase space of rapidity, azimuthal angle and transverse momentum. The measured cumulants are seen to have large non-zero values thus inferring strong genuine multiparticle correlations down to very small bin sizes. As was also concluded by OPAL

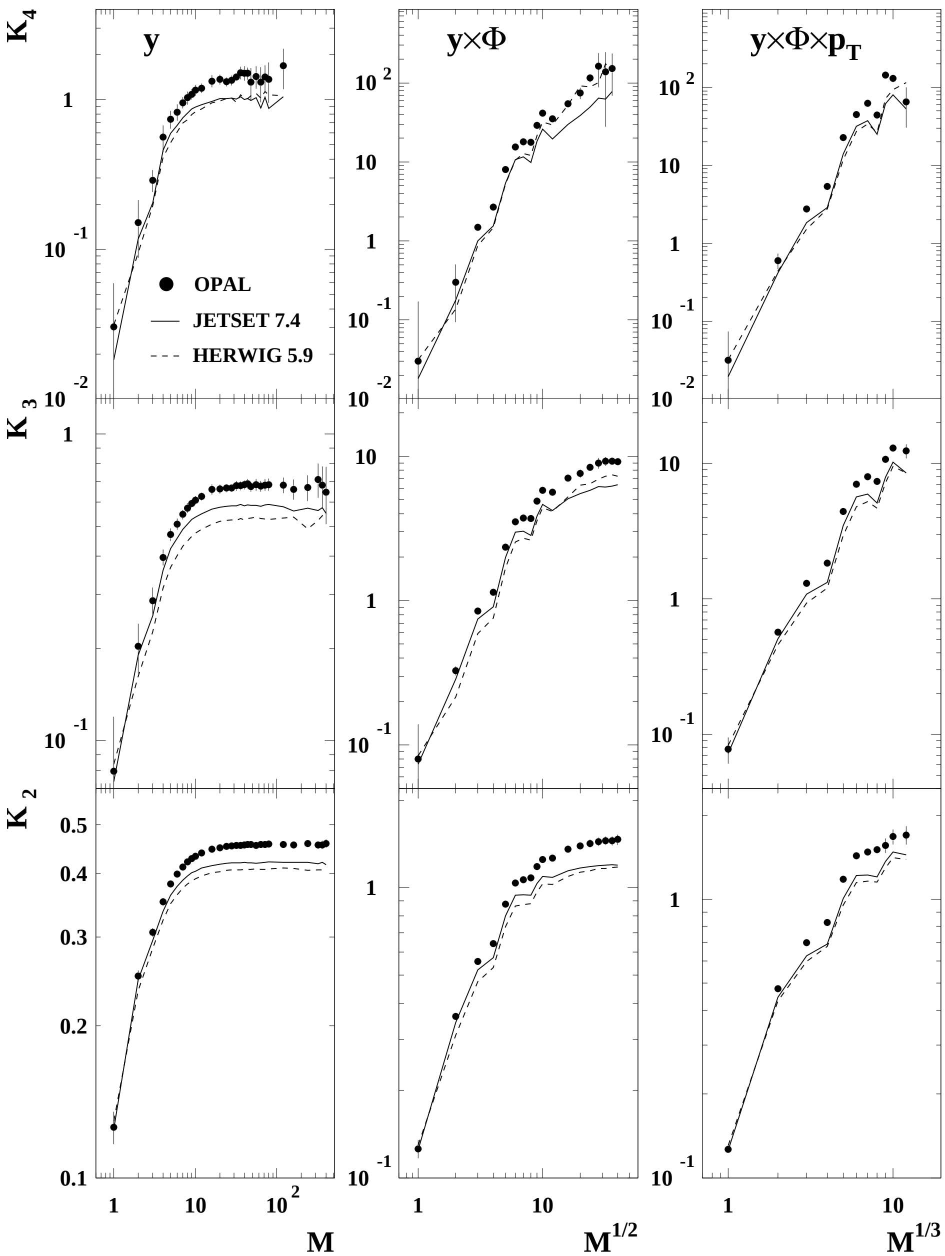

Figure 1. Cumulants of order $q = 2$, 3 and 4 as a function of $M^{1/D}$, where M is the number of bins of the D-dimensional sub-spaces of the phase space of rapidity (y), azimuthal angle (Φ) and transverse momentum (p_T), compared to two MC models. The data and the MC predictions are taken from Ref. [2].

[2], the high order fluctuations cannot be reproduced by lower-order correlations and indeed require also high-order correlations to be present. Thus higher order correlations do play an impor-

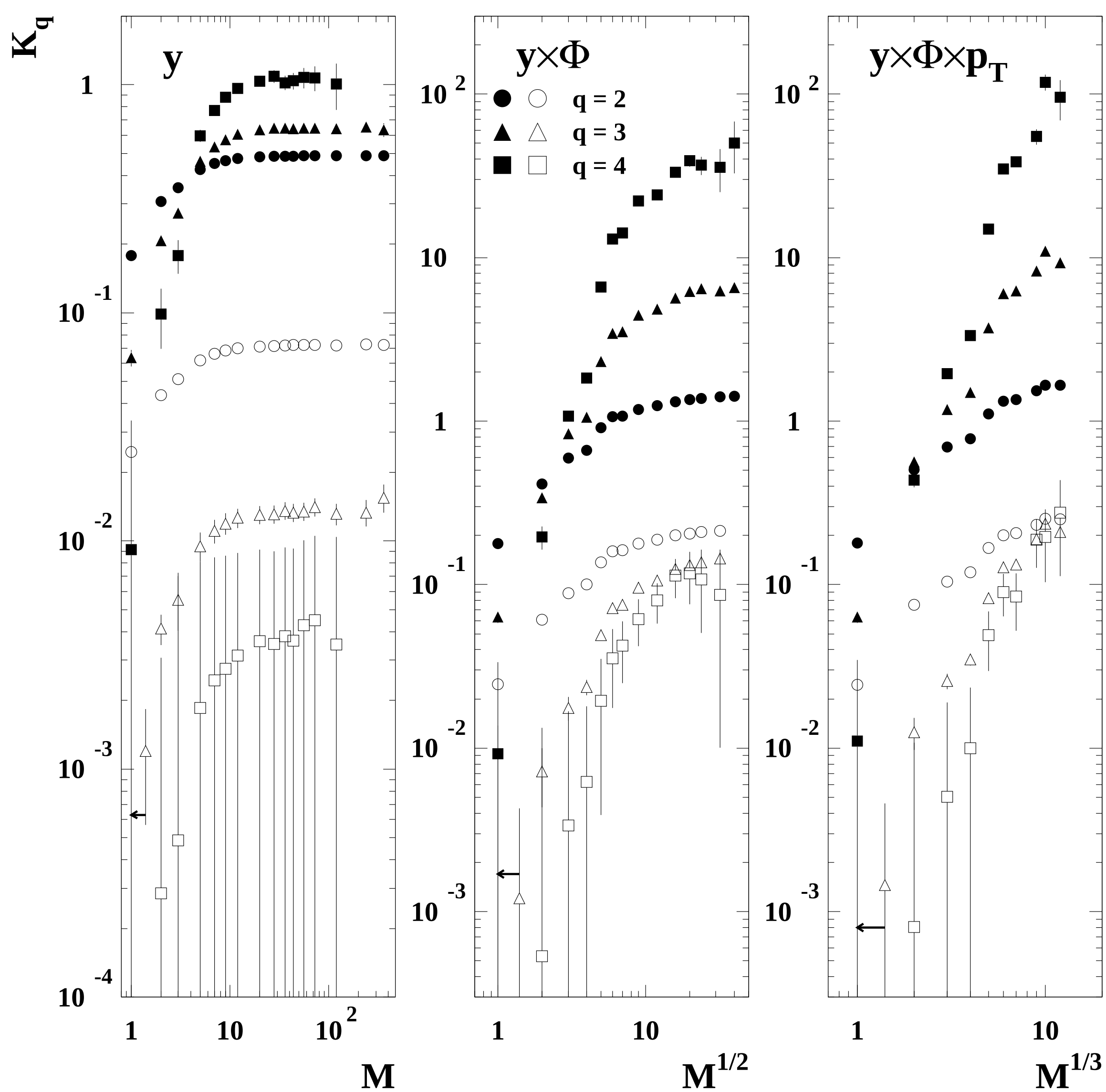

Figure 2. The MC predicted cumulants of order $q = 2$, 3 and 4 as a function of $M^{1/D}$, where M is the number of bins of the D-dimensional sub-spaces of the phase space of rapidity (y), azimuthal angle (Φ), and transverse momentum (p_T). The solid symbols represent the cumulants for a single source, while the open symbols are the cumulants values of seven sources.

tant role in the hadronic Z^0 decays processes.

As can be seen from Fig. 1, although the MC cumulants slightly underestimate the data starting at intermediate bin size, they reproduce well the over all behaviour of the cumulants as a function of the bin-size. This fact is utilised here for the study of the genuine multiparticle correlations dependence on the number of sources.

In Fig. 2 we compare the MC based cumulants of orders $q = 2$, 3 and 4 calculated from

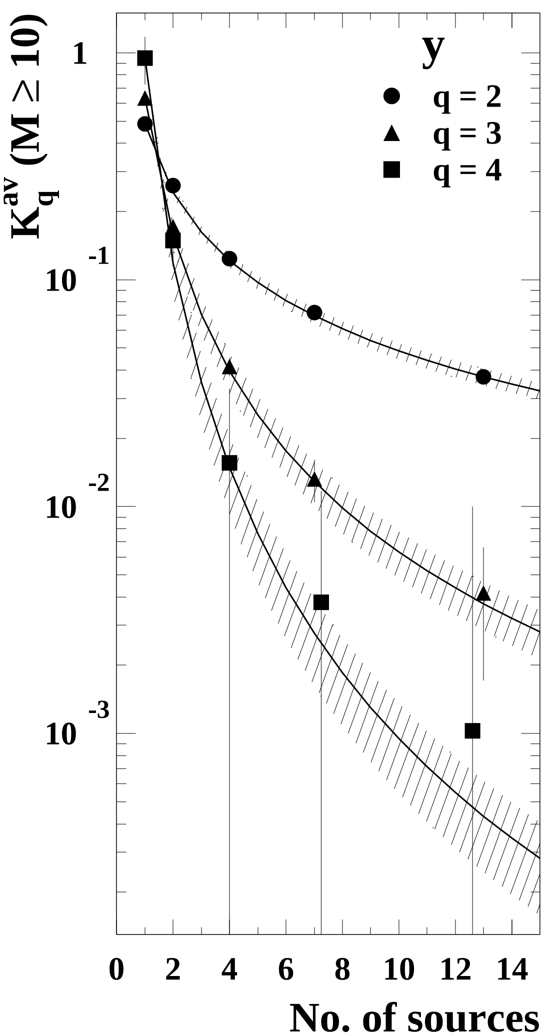

Figure 3. The dependence of the averaged 1-dimensional rapidity cumulants K_q^{av} of order $q = 2$, 3 and 4 on the number of e^+e^- sources. The cumulants were averaged over the M-range where they maintain an almost constant value (see Fig. 1). The lines represent the expected dilution according to Eq. (6) where q is neglected in comparison to the multiplicity n. The striped areas are the allowed regions when q is not neglected with respect to n (see text).

a single $e^+e^- \rightarrow Z^0 \rightarrow$ *hadrons* events (solid symbols) with those obtained by overlaying seven such events to represent seven hadronic sources (open symbols). The calculations were performed in the 3-dimensional phase space of rapidity, azimuthal angle and transverse momentum as well as in its projections onto 1-dimensional rapidity and onto 2-dimensional rapidity vs. azimuthal angle sub-spaces.

The following observations can clearly be made from Fig. 2.

- The existence of a dynamical component, i.e. rise of the cumulants with increasing number of bins M, is seen to be present both in the single source as well as in the case of many sources. Although the slopes of this scaling behaviour are smaller for several sources than for a single source, they are still strongly present. It is also evident that the scaling character is kept as the number of sources increases. For example, the single-source 1- and 2-dimensional cumulants level off at the same M values as those for seven sources. No such saturation exists for the one and seven sources cumulants in three dimensions.

- The genuine dynamical correlations, measured by the cumulants, significantly decrease as the number of sources increases. This decrease is stronger for higher order correlations. Whereas the two-particle cumulants suffer a reduction of an order of magnitude as the number of sources increases from one to seven, the four-particle cumulants diminish by three or four orders of magnitude.

- The hierarchy of the K_q cumulants is reversed as the number of sources increases. The cumulants derived from the single-source events increase with increasing q-order so that $K_2^{(1)} < K_3^{(1)} < K_4^{(1)}$, whereas the hierarchy in the cumulants calculated for seven sources is reversed namely, $K_2^{(7)} > K_3^{(7)} \geq K_4^{(7)}$. In addition, the multi-sources cumulants of order $q > 2$ have almost

the same reduced value namely, $K_3^{(7)} \approx K_4^{(7)} \lesssim \mathcal{O}(0.1)$. This last feature does not change as the dimension increases.

- The overall dominant feature of the analysis results is the diminishing value of the higher order cumulants as the sources number increases leaving the K_2 to be the dominant genuine multiparticle correlation.

To analyse further the observed diminishing effect, the cumulants were averaged over the M region where they are seen in Fig. 2 to reach an almost constant value, i.e. over $M \geq 10$ in one dimension (rapidity) and $M \geq 100$ in two dimensions (rapidity vs. azimuthal angle). The resulted M-averaged cumulants $K_q^{\rm av}$ are shown in Figs. 3 and 4 respectively for one and two dimensions. The corresponding values are also listed in Table 1 for the 1-dimensional cumulants and in Table 2 for the 2-dimensional ones together with the OPAL measured data cumulants [2] of single $\mathrm{e^+e^-} \to \mathrm{Z^0} \to$ *hadrons* events.

As Table 1 shows, the 1-dimensional data cumulants do agree with those derived from the MC sample for $q \leq 3$. Even though the single-source MC based $q = 4$ M-averaged cumulant lies lower than the measured points it is still consistent within errors with the data. This however is not the case for the two dimensions as can be deduced from the values given in Table 2. Already for $q \geq 3$ the measured and the MC based 2-dimensional averaged cumulants disagree. This is due the faster increase of the data cumulants compared to those obtained from MC (see Fig. 1).

From the features of the M-averaged cumulants shown in Figs. 3 and 4 one can conclude with the following obvious points.

- In one, as well as in two dimensions, the cumulants of order $q > 2$ decrease fast with the increased number of sources.

- In one dimension the hierarchy is reversed already for two sources and the two-particle correlations visibly dominate over the higher order ones.

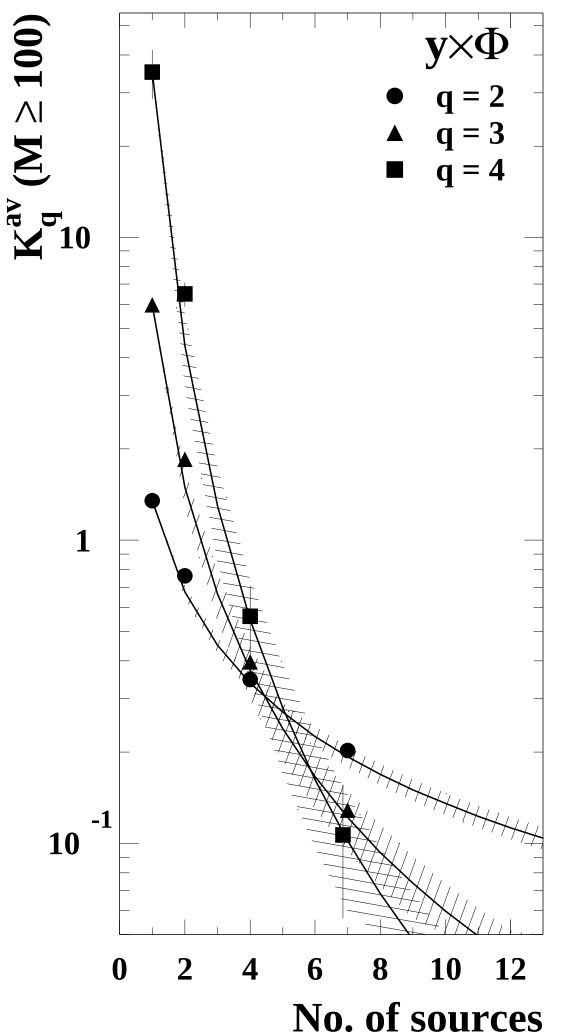

Figure 4. The same as Fig. 3 but for the 2-dimensional sub-space of rapidity and azimuthal angle.

- In two dimensions the hierarchy changes too, but at a higher number (> 2) of sources since additional correlations in the second variable (in our case the azimuthal angle) play also a role. The latter correlations are within a jet which are suppressed when all the phase space is projected into one direc-

Table 1
The Monte Carlo M averaged 1-dimensional rapidity $K_q^{\rm av}$ cumulants obtained in the $e^+e^- \to Z^0 \to$ *hadrons* reaction compared to those measured by OPAL [2] in single data events. The averages were taken over the M region where the cumulants reached a constant value (see Fig. 2).

No. of sources	$K_q^{\rm av}$ $(M \geq 10)$ $q=2$	$q=3$	$q=4$	Sample
	0.45 ± 0.01	0.67 ± 0.04	1.36 ± 0.21	OPAL data
1	0.486 ± 0.002	0.632 ± 0.017	0.950 ± 0.225	MC
2	0.260 ± 0.001	0.171 ± 0.006	0.147 ± 0.034	”
4	0.124 ± 0.001	0.041 ± 0.003	0.016 ± 0.012	”
7	0.072 ± 0.001	0.013 ± 0.003	0.003 ± 0.008	”
13	0.037 ± 0.001	0.004 ± 0.003	0.001 ± 0.009	”

Table 2
The Monte Carlo M averaged 2-dimensional (rapidity vs. azimuthal angle) $K_q^{\rm av}$ cumulants obtained in the $e^+e^- \to Z^0 \to$ *hadrons* reaction compared with those obtained in a recent OPAL measurements [2] of single data events. The averages were taken over the M region where the cumulants reached an almost constant value (see Fig. 2).

No. of sources	$K_q^{\rm av}$ $(M \geq 100)$ $q=2$	$q=3$	$q=4$	Sample
	1.39 ± 0.04	7.91 ± 0.36	100 ± 52	OPAL data
1	1.353 ± 0.005	6.0 ± 0.2	35.0 ± 6.5	MC
2	0.763 ± 0.002	1.85 ± 0.03	6.5 ± 0.6	”
4	0.347 ± 0.001	0.398 ± 0.012	0.6 ± 0.2	”
7	0.202 ± 0.001	0.129 ± 0.006	0.11 ± 0.05	”

tion, rapidity or azimuthal angle, as is discussed in [2]. Thus it seems that there are features in the second dimension, here the azimuthal angle, that lead to higher order , $q > 2$, correlations which do not contribute to the 1-dimensional rapidity correlations.

- At higher number of sources the dominant role of the two-particle correlations becomes more pronounced.

3.2. Correlation dilution due to source mixing

Within the procedure adopted here for the simulation of multi-source events, it is clear that if a genuine correlation exists it can only be detected in groups of q pions emerging from the very same source. In those q-group combinations which emerge from at least two sources, genuine correlations should not be present. This means that for K_q cumulants that are calculated over all possible q-pion groups, the higher the number of sources the more diluted will be the signal for genuine correlations.

For a given q-order the genuine correlation dilution factor is thus:

$$R_q \;=\; \frac{P_q^{\rm G}}{(P_q^{\rm G} + P_q^{\rm NG})}\ , \tag{4}$$

where $P_q^{\rm G}$ denotes the number of q-particle groups, e.g., pairs or triplets of pions, which emerge from the same source. The term $P_q^{\rm NG}$ stands for the number of all possible combinations of q-particle groups which emerge from at

least two sources.

Since all sources are produced in the same reaction and at the same energy, they do have an identical average charged multiplicity. For the estimation of R_q we assume that all the S sources have the same fixed charged multiplicity n. In this case one has $P_q^{\mathrm{G}} = S\binom{n}{q}$, and the dilution factors at $q = 2$, 3 and 4 are given by

$$R_2 = \frac{\binom{n}{2} S}{\binom{n}{2} S + n^2\binom{S}{2}} \xrightarrow{n\gg 1} \frac{1}{S} ,$$

$$R_3 = \frac{\binom{n}{3} S}{\binom{n}{3} S + n^3\binom{S}{3} + 2\,n\binom{n}{2}\binom{S}{2}} \xrightarrow{n\gg 2} \frac{1}{S^{\,2}} ,$$

$$R_4 = \frac{\binom{n}{4} S}{\binom{n}{4} S + n^4\binom{S}{4} + 3\,n^2\binom{n}{2}\binom{S}{3} + \binom{n}{2}^2\binom{S}{2} + 2\,n\binom{n}{3}\binom{S}{2}} \xrightarrow{n\gg 3} \frac{1}{S^{\,3}} , \qquad (5)$$

where the denominators include the number of all possible q-particle combinations in S sources of charged multiplicity n. These dilution factors dependence on the number of sources can also be derived in terms of cumulants [10].

From these R_q relationes follows that as long as $n \gg q$ one has a general expression for the dilution factor namely,

$$R_q \xrightarrow{n\gg q} \frac{1}{S^{\,q-1}} . \qquad (6)$$

To compare the dilution factors R_q with our correlation results shown in Figs. 3 and 4, they do have to be multiplied by $K_q^{\mathrm{av}(1)}$ which is a measure of the genuine q-order correlation present in a single sources. The solid lines shown in Figs. 3 and 4 thus represent the diluted cumulants $K_q^{\mathrm{av}} = K_q^{\mathrm{av}(1)} \times R_q$. The striped areas in which the lines are embedded are the allowed regions when q is not neglected with respect to the multiplicity n.

The agreement between the cumulant calculations and the dilution factors predictions is really remarkable. In the one-dimensional case the predictions follow the MC based cumulants for $q = 2$ and 3 and are certainly well within the rather large errors of the $q = 4$ cumulants. In two dimensions, in addition to the remarkably good agreement with the MC cumulants, the dilution factor predictions follow closely also the hierarchy change of the K_q moments.

For the order $q = 2$ one can relax the fixed charged multiplicity assumption and allow them to be different and still retain the $R_q \simeq 1/S^{q-1}$ relation as long as the multiplicity distribution is of a Poisson nature. This however is not the case for orders higher than 2. Nevertheless for order $q = 3$ the relation $R_3 = 1/S^2$, derived from the fixed multiplicity assumption, is still valid as it describes well the K_3^{av} values up to at least thirteen sources (see Fig. 3). The large cumulants' errors associated with the $q = 4$ order prohibits to judge how accurate is the $R_4 = 1/S^3$ relation.

An additional interesting and useful application of the relation $R_q \simeq 1/S^{q-1}$ is that it offer a method to estimate the average number of sources $\langle S\rangle$ via the cumulant averaged values over the large M region of two sequential q-orders through the ratio,

$$\langle S\rangle \simeq \frac{K_{q+1}^{\mathrm{av}(1)}}{K_q^{\mathrm{av}(1)}} \times \frac{K_q^{\mathrm{av}}}{K_{q+1}^{\mathrm{av}}} . \qquad (7)$$

3.3. Comparison with hadron and nucleus induced reactions

As is already mentioned in the introduction, the genuine correlations measured in e^+e^- annihilations [2] are found to be weaker in hadronic interactions [3–5, 13] and even more so in nuclear collisions [3, 6–9]. In nucleus-nucleus collisions at ultra-relativistic energies only the second-order correlations are found to have non-zero values in rapidity [3, 7–9], while in two dimensions (rapidity and azimuthal angle) the third-order cumulants are also detected [6, 9].

In Table 3 we list the results obtained by several experiments [3, 6, 8, 9] on the M-averaged rapidity cumulant values for $q = 2$ and 3. In Table 4 the analogous cumulants for $q = 2$, 3 and 4 are given as measured in two dimensions, rapidity vs. azimuthal angle [6, 9]. These average values were taken over the M-region where the

Table 3
The M averaged 1-dimensional rapidity cumulants $K_q^{\rm av}$ of orders $q = 2$ and 3 measured in several hadronic reactions. The cumulants were averaged over the M regions where they were seen to approach a constant value. The quoted values were estimated from the relevant published figures in the papers listed in the table references.

$\langle n_{\rm ch} \rangle$	$K_q^{\rm av}$ $q=2$	$K_q^{\rm av}$ $q=3$	Reaction	Beam energy (GeV)	Ref.
~ 8	0.32 ± 0.02	0.26 ± 0.12	πp	250	[3]*
21.1	0.21 ± 0.04	0.14 ± 0.18	pEm	200	[3]*
> 50	0.34 ± 0.02	0.12 ± 0.03	AuEm	$10.6A$	[6]
73.3	0.21 ± 0.03	0.05 ± 0.07	SiEm	$14.5A$	[6, 9]
81.1	0.20 ± 0.05	0.00 ± 0.12	OEm	$60A$	[9]
154.9	0.11 ± 0.05	0.01 ± 0.18	OEm	$200A$	[3]*
216.1	0.25 ± 0.05	0.08 ± 0.10	SEm	$200A$	[9]
272.6	0.09 ± 0.05	0.00 ± 0.18	SEm	$200A$	[3]*
289.8	0.11 ± 0.08	0.02 ± 0.10	SAu	$200A$	[8]**
355.0	0.08 ± 0.05	0.00 ± 0.08	SAu	$200A$	[3]*
383.9	0.07 ± 0.04	0.00 ± 0.05	SAu	$200A$	[8]***

* Calculations are based on the measured factorial moments. ** Semi-central collisions. *** Central collisions.

Table 4
The M averaged 2-dimensional (rapidity vs. azimuthal angle) cumulants $K_q^{\rm av}$ of orders $q = 2$, 3 and 4 measured in nucleus-nucleus collisions. The cumulants were averaged over the M regions where they were seen to approach a constant value. These quoted values were estimated from the relevant published figures given in the references listed in the table.

$\langle n_{\rm ch} \rangle$	$K_q^{\rm av}$ $q=2$	$K_q^{\rm av}$ $q=3$	$K_q^{\rm av}$ $q=4$	Reaction	Beam energy (GeV)	Ref.
> 50	0.79 ± 0.02	1.3 ± 0.1	3.2 ± 2.8	AuEm	$10.6A$	[6]
73.3	0.20 ± 0.05	0.09 ± 0.03	0.4 ± 0.3	SiEm	$14.5A$	[6]
119.3	0.57 ± 0.03	0.3 ± 0.1	0.4 ± 0.7	OEm	$200A$	[9]
216.1	0.25 ± 0.03	0.07 ± 0.04	0.1 ± 0.2	SEm	$200A$	[9]

cumulants are seen in the published figures to reach a constant level. The reactions and their cumulants values are ordered according to their reported mean charged multiplicity, from the lowest value to the highest one.

Tables 3 and 4 show that in hadron including nucleus induced reactions the two-particle correlations decrease rather fast as the mean multiplicity increases both in one and two dimensions. However the three-particle correlations are found to be essentially non-existing in one dimension even at moderately small mean multiplicity, while in two dimensions these correlations are seen still to be non-zero. Moreover, the higher the order of the 2-dimensional correlation the larger the value is. These measurements, particularly for 2-dimensional cumulants are in an amazing agreement with our findings, see Table 2 and Fig. 4.

Notwithstanding the possibility that production of hadrons in e^+e^- annihilation may well be simpler than in hadron induced reactions, it may nevertheless be instructive to relate our results to the measured correlation data listed in Tables 3 and 4. Inasmuch that the mean multiplicity increases with the number of sources, the measurements consistent with our findings. As well as in nucleus induced reactions, we found the decrease in the two-particle rapidity correlations and the

absence of three-pion rapidity correlations. In two dimensions our results show a surviving of higher order correlations and a change of the hierarchy not starting from two sources, but later at larger number of them, the effects which one can see in the data in nucleus-nucleus reactions. All this demonstrates the effect of dilution of the correlations with increased number of sources.

A quantitative comparison between our findings and the correlations in nucleus-nucleus and hadron induced reactions is hard to make mainly because of the lack of information on the values of $K_q^{\mathrm{av}(1)}$ which should be derived from nucleon-nucleon collisions data. In addition in many of the existing cumulant data of the nucleus induced reactions have still too large errors.

Recently the two-pion BEC have been studied [22] in $\bar{\mathrm{p}}\mathrm{p}$ reaction at centre of mass energy of 630 GeV as a function of multiplicity by using the normalised cumulants method similar to the one used here. In that analysis it has been found that the correlations of the cumulants of the like-sign pions as well as the opposite-sign pions decrease with the multiplicity n_{ch}. From our analysis we expect the pair correlation to decrease as $1/S$, where S is the number of sources. This indicates that indeed the multiplicity is at least partially proportional to S. The BEC dependence has also been investigated in the framework of the totally coherent emission picture [29] and in the quantum optical approach [21] where the conclusions were that these correlations are weaker as the multiplicity increases.

4. SUMMARY AND CONCLUSIONS

In the present work we investigated the effect of many emission sources on the genuine correlations in mutihadron final state. To this end we adopted a procedure which should minimise the confusion introduced by other variables like charged multiplicity. For the genuine correlations measurement we utilised the normalised cumulant method. To simulate the situation of many sources event we did overlay Monte Carlo generated hadronic Z^0 events treating them as one event. This JETSET 7.4 MC sample of some five million events, tuned to the OPAL data taken at LEP1 on the Z^o mass, has previously described rather well the measured correlations in the $e^+e^- \rightarrow Z^0 \rightarrow$ *hadrons* data. We studied the cumulants in one, rapidity, dimension and in two dimensions of rapidity and azimuthal angle.

The results shown here demonstrate that the cumulants, obtained from a single-source events and from events of many sources, almost do not change their basic structure with the decrease of the width of phase-space bins. This means that the scaling is preserved although larger slopes are seen to be in the case of one source compared with those for several sources.

Due to source mixing the higher order cumulants are suppressed both the 1-dimensional (rapidity) and the 2-dimensional (rapidity $\times$ azimuthal angle) and diminish to zero as the number of sources increases. In both case as S increases the hierarchy is reversed and the cumulant of the lower order $q = 2$ dominates. This happens in the one dimension case already when $S = 2$ whereas in the two dimension occurs only at a higher S values of around $S = 5$.

The correlations observed are very well reproduced by assuming that genuine correlations of the order q can only exist when all the q hadrons are emerging from the same source. Therefore the dilution of the genuine correlation signal is proportional to the ratio of the probability that the q hadrons will emerge from the very same source. From simple combinatorial considerations this probability can then be approximated by $1/S^{q-1}$. Thus a measurement of the correlations of two sequential orders in q can be used to estimate the average number of sources.

The genuine correlations measured in hadron and nucleus induced reactions do follow qualitatively the findings of our work. In particular in nucleus-nucleus reactions, where many sources are expected to contribute to the final hadronic state, the rapidity cumulants of the $q > 2$ orders are very small and indeed consistent with zero. The $q = 2$ order of the 1-dimensional cumulants still survive but they also are getting smaller as the atomic number of the nuclei increases. In two dimensions, the cumulants are still different from zero even at the $q = 4$ order, however their value decreases and a change in the

hierarchy takes place as the multiplicity increases. The general observation that cumulants decrease as the multiplicity increases reassures the common notion that the higher the multiplicity the larger the number of sources.

Our results may also be useful for the understanding of other types of measured correlations like that obtained from the Bose-Einstein interferometry of two and more identical bosons. It has been previously pointed out [30] that in the absence of final state interactions the BEC of the $e^+e^- \to W^+W^- \to$ *hadrons* will be half of that of the Z^0 decay to hadrons. From our study it follows directly that the two-particle BEC, or any other correlations, in the two-source reaction $e^+e^- \to W^+W^- \to$ *hadrons* should be reduced by a factor two as compared to that of the hadrons emerging from one W-boson.

Acknowledgements

We would like to thank the organisers of the Workshop on Multiparticle Production for their very kind hospitality and for the warm and pleasant atmosphere that they maintained over the whole conference duration.

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